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Taper equation compatible with volume equation for the Hungarian oak stands under restoration at Northern Greece

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Abstract

For the coppice stands of Hungarian oak (*Quercus frainetto* Ten., also known as *Quercus conferta* Kit.) at the forest of Lofos of Pieria prefecture (Northern Greece), linear and nonlinear taper equations were compared, which estimate the diameter d_{h_i} at any tree height h_i. For this aim a systematic sample of 90 trees was selected. At each tree the stump height diameter, the diameter at 80 cm height and the breast height diameter were measured. Then diameters per 2 m above the breast height diameter were estimated, that is at 3.3, 5.3, 7.3… m above the ground and also total tree heights. For the comparison of equations that was fitted to data, the residuals normality, their randomness, independence, homogeneity of variance and the value of mean absolute error were examined. In order to test the compatibility of the selected equation with the corresponding volume equation for the region, the homogeneity test of Wilcoxon was applied and it was found that the two equations are compatible.

Key words: Homogeneity tests, regression analysis, residuals analysis, taper equations, volume equations, volume tables.

Introduction

Taper equations were developed in order to predict the diameter d_{h_i} at any tree height h_i. The tree diameter is generally decreased from the base to the top. The reduction pattern determines the trunk form [23]. The comprehension of the trunk form allows better estimates of the tree volume or biomass, better estimates for the types and quantities of various wood products and better comprehension of competition and conditions of trees increase and it depends on the tree type, diameter, stands density, age and site quality [12].

As far as taper equations fitted to oak data, Hilt [11] studied the form of 5 oak species (*Quercus alba*, *Q. prinus*, *Q. velutina*, *Q. coccinea* and *Q. rubra*) in USA by modifying the equation given by Bruce *et al.* [2]. Thomas and Parresol [27] fitted a trigonometric equation to oak data (*Quercus phellos*) in Mississippi. Trincado *et al.* [28] compared the equations given by Kozak *et al.* [18], Demaerschalk [4], Biging [1] and Riemer *et al.* [24] for *Quercus robur* in Germany.

A taper equation compatible with a volume equation for two oak species (*Quercus robur* and *Q. petraea*) in Denmark was developed by Tarp-Johansen *et al.* [26]. Cakir and Nix [3] studied the effect of early logging in the trunk form in various stands and most of them contained important oak species (*Quercus alba*, *Q. velutina*, *Q. coccinea*, *Q. falcata*), in North Carolina. In Greece, taper equations are available for *Quercus conferta* at Chalkidiki [14, 15].

If a taper equation is available, we can calculate the volume v_i of any part of the tree, between heights h_1 and h_2, by integrating the equation as follows:

$$v_i = \frac{\pi}{4}\int_{h1}^{h2} d_h^2\,dh$$

We can also calculate the total tree volume v by replacing the integration limits h_1 and h_2 with zero and total tree height H respectively, that is:

$$v = \frac{\pi}{4}\int_{0}^{H} d_h^2\,dh$$

A lot of researchers dealt with the fitting of taper equations, which are compatible with volume equations, for the whole trunk [4, 6-8, 10, 26] or for different parts of the trunk [7, 25].

A taper equation, which is compatible with a volume equation, when it is integrated, with integration limits $h_1 = 0$ and $h_2 = H$ gives the same total tree volume as the volume given by the compatible volume equation.

In Greece *et al.* [16] concluded that the integration of the taper equation for oak stands in Chalkidiki gives bigger volumes, compared to the volumes given by the volume equation, and constitutes a more accurate method of volume estimation.

The aim of this work was to 1) compare taper equations and select the most appropriate one for Hungarian oak (*Quercus frainetto* Ten., also known as *Quercus conferta* Kit.) in the forest of Lofos (Pieria prefecture, Northern Greece) and 2) check the compatibility with the corresponding volume equation. The forest has an area of 998 ha [9].

Materials and Methods

For the data collection systematic sampling was used, which is preferred in the forestry practice from the simple random sampling, because systematic surfaces are easier to locate. Beforehand a lower diameter limit was determined (17 cm) and the trees entered in the sample had breast height diameter bigger than this limit. This was done because the small trees are usually many in a forest, while their contribution in basic forest variables (volume and others) is small [19]. In a forest map of Lofos 8 parallel lines 700 km apart were drawn. A tree was measured every 200 m at length of the parallels. The sample that resulted had a size of 90 trees of Hungarian oak (*Quercus frainetto* Ten., also known as *Quercus conferta* Kit.). In these trees the stump height diameter ($h_i = 0.30$ m), the diameter at height $h_i = 0.80$ m, the breast height diameter D ($h_i = 1.30$ m), the diameters per 2 m above breast height diameter, that is in 3.3, 5.3, … m above the ground and the total tree height H were measured. In order to fit taper equations to data, following 11 linear and nonlinear regression equations were tested [14]:

$$d_{h_i} = b_0 + b_1 h_i \quad (1)$$

$$d_{h_i} = b_1 \left(H - h_i \right) \quad (2)$$

$$d_{h_i} = b_0 + b_1 \frac{h_i}{H} + b_2 D + b_3 H \quad (3)$$

$$d_{h_i} = D \left(\frac{1.3}{h_i} \right)^{\alpha_1} \quad (4)$$

$$d_{h_i} = D \left(\frac{H - h_i}{H - 1.3} \right)^{\alpha_1} \quad (5)$$

$$d_{h_i} = b_1 D \left| \frac{H - h_i}{H - 1.3} \right|^{\alpha_1} \quad (6)$$

$$d_{h_i} = b_1 D^{\alpha_1} \left(H - h_i \right)^{\alpha_2} H^{\alpha_3} \quad (7)$$

$$\frac{d_{h_i}}{D} = b_0 + b_1 \frac{h_i}{H} \quad (8)$$

$$\frac{d_{h_i}}{D} = b_1 \frac{H - h_i}{H} + b_2 \frac{\left(H^2 - h_i^2 \right) h_i}{H^2 - D} \quad (9)$$

$$\frac{d_{h_i}}{D} = b_1 D \frac{H - h_i}{H - 1.3} + b_2 \frac{\left(H^2 - h_i^2 \right)\left(h_i - 1.3 \right)}{H^2} \quad (10)$$

$$\frac{d_{h_i}}{D} = \sqrt[k]{\frac{H - h_i}{H - 1.3}} \text{ with } k = b_1 + b_2 \left(\frac{H - h_i}{H - 1.3} \right)^6 + b_3 \frac{D}{H} + b_4 \left(\frac{H - h_i}{H - 1.3} \right)^2 \frac{D}{H} \quad (11)$$

where d_{h_i} = tree diameter at height h_i (m), D = breast height diameter (m), H = total tree height (m), a_i, b_i = regression coefficients.

Regression analysis was applied with the use of the SPSS statistical software [17, 22]. In order to compare the fitted equations the residuals normality (Lilliefors test), randomness (runs test), independence (chi-square test), homogeneity of variance (Levene test) and arithmetic mean were checked [20, 21].

After the selection of the most appropriate taper equation, the homogeneity of the volumes calculated from the integration of the taper equation and the volumes of the corresponding volume equation were tested. For a selected taper equation that estimates d_{h_i} in function to D, the corresponding volume equation is ln$v$ = 1.418-0.523/D. For a selected taper equation that estimates d_{h_i} in function to D and H, the corresponding volume equation is ln$v$ = 3.826-48.921/H-0.02551H/D [13]. In order to test the homogeneity of the volumes the Wilcoxon signed ranks test was applied. It is a non-parametric test, which was applied instead of the t-test, because there was no information about the normality of the original population. The volumes of the integrated taper equation constitute the first group of values and the volumes of the volume equation the second group. If these groups of values are homogeneous, then the taper equation is compatible with the volume equation.

Results and Discussion

The summary statistics for the sample of the 90 trees are the breast height diameter having a range from 18 to 36 cm, mean 24 cm and standard deviation 5 cm and the total tree height having a range from 16 to 25 cm, mean 19.2 m and standard deviation 1.94 m.

The statistical tests for the 11 equations are given in Table 1. According to these results, the equation that fulfills all 5 criteria (residuals normality, randomness, independence, homogeneity of variance for a 1% probability and zero mean) is the 3rd one:

$$d_{h_i} = b_0 + b_1 \frac{h_i}{H} + b_2 D + b_3 H$$

In Table 1, the acceptable criteria values are highlighted. The selected taper equation has the following values for the regression coefficients: $b_0 = 0.189$, $b_1 = -0.182$, $b_2 = 0.612$ and $b_3 = -0.00399$, with confidence intervals [0.169, 0.210], [-0.191, -0.173], [0.566, 0.658] and [-0.005, -0.003], respectively, coefficient of determination $R^2 = 0.899$ and standard error of estimation $s_e = 0.164$.

In order to check the compatibility of the selected taper equation with the corresponding volume equation, the volume of the sample trees was calculated with 2 methods: The first method is the volume equation ln$v$ = 3.826-48.921/H-0.02551H/D [13]. The second method is the integration of the selected taper equation:

$$d_{h_i} = 0.189 - 0.182 \frac{h_i}{H} + 0.612D - 0.00399H$$

as follows: $v = \frac{\pi}{4} \int_0^H d_h^2 \partial h =$

$$\frac{-\pi H}{4 \times 0.182} \frac{\left(0.189 - 0.182 + 0.612D - 0.00399H\right)^3 - \left(0.189 + 0.612D - 0.00399H\right)^3}{3}$$

The volumes of the first method had a range from 0.2233 m^3 to 0.9634 m^3, mean 0.4786 m^3 and standard deviation 0.2115 m^3. The volumes of the second method had a range from 0.2964 m^3 to 0.9370 m^3, mean 0.4905 m^3 and standard deviation 0.1703 m^3.

The Z-criterion significance is 0.016 (Table 2), which means that both values groups do not differ statistically significantly, for a 1% probability. Therefore, the two equations (volume and taper) are compatible.

Table 1. Residuals tests.

Statistic	Significance of D (normality)	Significance of Z (randomness)	Significance of χ_P^2 (independence)	Significance of L (homogeneity of variance)	Mean absolute residual $\sum_{i=1}^{n} \frac{\lvert d_{h_i} - \hat{d}_{h_i} \rvert}{n}$
Acceptable value (probability 1%)	>0.01	>0.01	>0.01	>0.01	≈ 0
Equation 1	0.000	0.011	0.183	0.079	0.0000
Equation 2	0.033	0.000	0.918	0.024	0.0191
Equation 3	0.024	0.128	0.316	0.164	0.0000
Equation 4	0.000	0.000	0.538	0.698	0.0031
Equation 5	0.001	0.013	0.316	0.693	0.0013
Equation 6	0.010	0.006	0.316	0.837	0.0014
Equation 7	0.023	0.584	0.316	0.707	0.0005
Equation 8	0.200	0.002	0.347	0.914	0.0008
Equation 9	0.000	0.045	0.316	0.432	0.0059
Equation 10	0.074	0.006	0.316	0.652	0.0041
Equation 11	0.200	0.045	0.316	0.783	0.0012

Table 2. Homogeneity test.

	Number	Mean rank	Sum of ranks	Z	-2.416
Rank of positive difference (volume equation < integrated taper equation)	58	45.66	2648.00	Asymptotic significance (2-tailed)	0.016
Rank of negative difference (volume equation > integrated taper equation)	32	45.22	1447.00		
Zero differences (volume equation = integrated taper equation)	0				
Total	90				

Conclusions

The comparison of 11 taper equations for Hungarian oak at the forest of Lofos of Pieria prefecture (Northern Greece) showed that the most appropriate is $d_{h_i} = 0.189 - 0.182\frac{h_i}{H} + 0.612D - 0.00399H$.

Total tree volumes estimated from integration of this equation are calculated by the formula:

$$v = \frac{-\pi H}{4 \times 0.182} \frac{(0.189 - 0.182 + 0.612D - 0.00399H)^3 - (0.189 + 0.612D - 0.00399H)^3}{3}$$

Homogeneity test between two dependent groups of values (volumes of the sample trees calculated from the integration of the taper equation and volumes from the volume equation $\ln v = 3.826 - 48.921/H - 0.02551H/D$) showed that the two groups do not differ statistically significantly. Therefore, the selected taper equation is compatible with the volume equation.

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